独占的公企業の公共料金基準

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"A crucial point regarding public utilities is shown in the technical conditions of production and supply which introduce service markets into partial or perfect monopoly." (Kita, K.; "Theory of public utilities." 1961.)

The so called "Monopoly of public utilities" is a 'necessary evil' and a 'substitute' for desirable framework. Local monopoly of public utilities is guaranteed legally, but regulated from various aspects.

A "Fair Rate of Return" criterion is one of them. Under this regulation, a monopoly must set prices not to violate the ratio of "net revenue" per value of capital stock, whose upper limit is set by the regulatory agency.

Averch and Johnson[1*] arrived at the conclusion in their paper that monopoly structure tends to invest more under the fair rate of return criterion than under no regulation condition and that if the firm can enter several markets the firm can earn more profits (even with losses) in some of the markets.[1*]

The implication is that the firm operating in oligopolistic second markets may have an advantage over competing firms. The regulated firm can "afford" to take (long-run) losses in these second markets while competing firms cannot operate.

Under these circumstances, monopoly firm may succeed in either driving lower efficiency firms out of these markets or discouraging their entry into them.

Even in the case of a long–run loss, the regulated firm finds his operations in such markets to be advantageous as long as the firm is allowed to keep up its capital input in these markets within its rate base; i. e. the monopoly can satisfy the constraints with more profits in multi–market than in the single market (case).

In this report, we analyse how monopoly seeks for profit–maximizing, in the context of regulation and social welfaire.

Suppose a monopoly produces an output Y with inputs K (capital), and L (labor), *i.e.* $Y=F(K, L)$, $Y, K, L \geqq 0$. Y, K, L are of course measured in flow.

Here, we assume $\frac{\partial F}{\partial K}=F'_K \gtrless 0=$ ……(1) $\frac{\partial F}{\partial L}=F'_l \gtrless 0$ ……(2)

$F''_K \angle 0 \quad F''_l \angle 0$ ……(3) Where, F''_K means $\frac{d^2F}{dK^2}$

$F(0, 0)=F(0, L)=F(K, 0)=0$ ……(4)

We can assume $F''_K \gtrless 0$, $F''_l \gtrless 0$ under some conditions.

If F″(both) imply the concavity of $P(Y) \cdot F(K, L)$, where $P(Y)$ is the inverse demand function, such that $\frac{dP(Y)}{dY} \angle 0$; we can only use this assumption.

But as Takayama[2*] comments, concavity of $F(K, L)$, … which is repre sented by F''(both k, l)$\angle 0$ …… dose not necessarily imply the concavity $P(Y)$, $F(K, L)$.[2*]

So, we now assume that $P(Y)$, $F(K, L)$ are concave. Then, the profits adapt concave function.

Now, we define $\pi=P(Y) \cdot Y-rK-wL$ ……(5)

And fair rate of return s as $\dfrac{P(Y)\cdot Y-wL}{cK}\leqq s$ ……(6)

(Above mentioned r and w are the prices of K and L respectively, we for simplicity, assume them to be constant. s−r is a profit/investment.

(per unit)

The numerator is the "net revenue", and the denominator is the value of capital stock. By changing the measure of K, we can make c=1.

(In this paper, we discuss only one term, or static case. So, by letting c=1, we have the constraint as follows.)

$$P(Y)\cdot Y-sK-wL\leqq 0$$

Since, $\pi=P(Y)\cdot Y-rK-wL=P(Y)\cdot Y-sK-wL+(s-r)K\leqq(s-r)K,\ s\gneqq r$ must hold for the firm to produce positive amount of output.

We now assume that the firm seekes for profit maximizing under the constraint (7).

To define the Lagrangian as,

$$Q(K,\ L,\lambda)=P(Y)\cdot Y-rK-wL-\lambda[P(Y)\cdot Y-sK-wL].$$

If the constraint is a concave function, the Kuhn−Tucker−Ragrange conditions are necessary and sufficient for the maximum of π.

The Kuhn−Tucker necessary conditions for a maximum of π at K*, L*, λ* are: ……

$$\pi^*=\frac{\partial Q(K^*,\ L^*,\ \lambda^*)}{\partial K}=(1-\lambda^*)\ [P(Y)+\frac{dP(Y)}{dY}\cdot Y]F_K'+(\lambda^*s-r)\leqq 0.$$

$\pi^*\angle 0$ implies $K^*=0$, *for* $K^*\geqq 0$.

$$\pi^*=\frac{\partial Q(K^*,\ L^*,\ \lambda^*)}{\partial L}=(1-\lambda^*)\ [P(Y)+\frac{dP(Y)}{dY}\cdot Y]F_l'+(\lambda-1)w\leqq 0.$$

$\pi^*\angle 0$ implies $L^*=0$, *for* $L^*\geqq 0$.

$$\pi^*=\frac{\partial Q(K^*,\ L^*,\ \lambda^*)}{\partial \lambda}=P(Y)\cdot Y-sK^*-wL^*\leqq 0 \quad for\ \lambda^*\geqq 0.$$

$\pi^*\angle 0$ implies $\lambda^*=0$,

We have analyse the case of $K^* \gtrsim 0$ and $L^* \gtrsim 0$. By rewriting the case of λ as $\lambda^*\ [P(Y)\cdot Y - sK^* - wL^*]=0$, we have the following conditions.

Let $Y=F$, and $\dfrac{dP(Y)}{dY}=P'$

$(1-\lambda^*)\cdot(P+P'F)\cdot F'_K = r-\lambda^* s$　　(8) − a

$[(P+P'F)\cdot F'_l - w]\cdot(1-\lambda^*)=0$　　(8) − b

$\lambda^*\cdot(P\cdot F - sK^* - wL^*)=0$　　(8) − c

$\lambda^* \geqq 0$　　(8) − d

If there is no constraint, the firm maximizes,

$$\pi = P\cdot F - rK - wL.$$

The necessary condition for maximization is,

$$\frac{\partial \pi}{\partial K} = (P+P'F)\cdot F'_K - r = 0 \qquad \text{and}$$

$$\frac{\partial \pi}{\partial L} = (P+P'F)\cdot F'_l - w = 0 \qquad \text{They imply} \cdots$$

$\dfrac{F'_K}{F'_l} = \dfrac{r}{w}$ As is well known, the corresponding cost is minimized.

Up to this stage, we assumed that the constraint was a concave function. yet, even π was a convex function, the paper by Arrow–Hurwicz–Uzawa states that the convexity guarantees the necessity of the constraint maximum.

Now then, from (8) −a and b,
we have

$$\frac{F'_K}{F'_l} = \frac{r-\lambda^*\cdot s}{(1-\lambda^*)w} = \frac{(1-\lambda^*)r - \lambda^*(s-r)}{(1-\lambda^*)w} = \frac{r}{w} - \frac{\lambda^*(s-r)}{(1-\lambda^*)w} \qquad \cdots(9)$$

As is shown above, when the administrative regulation is performed along the "fair rate of return" criterion, It deviates some $\dfrac{\lambda^*(s-r)}{w(1-\lambda^*)}$.

It also means, whether numerator or denominator of marginal rate of substitution $\frac{F_K}{F_l}$ is larger or smaller, and vice versa, either L or K is spent wastefully.

Since P • F is assumed to be concave, the following property must be satisfied: …

$$P[F(0, 0)]\cdot F(0, 0)-P[F(K^0, L^0)]\cdot F(K^0, L^0) \leqq \frac{\partial P[F(K^0, L^0)]\cdot F(K^0, L^0)}{\partial K}\cdot(0-K)$$

also,

$$P[F(0, 0)]\cdot F(0, 0)-P[F(K^0, L^0)]\cdot F(K^0, L^0) \leqq \frac{\partial P[F(K^0, L^0)]\cdot F(K^0, L^0)}{\partial L}\cdot(0-L)$$

for any $K, L \geqq 0$.

Let $K=K^* \gneqq 0$ *and* $L=L^* \gneqq 0$. Then, we have …

$$P[F(0, 0)]F(0, 0)-P[F(K^*, L^*)]\cdot F(K^*, L^*) \leqq \frac{\partial P[F(K^*, L^*)]\cdot F(K^*, L^*)}{\partial K}\cdot(0-K^*) + \frac{\partial P[F(K^*, L^*)]\cdot F(K^*, L^*)}{\partial L}\cdot(0-L^*)$$

Now recall $F(0, 0)=0$, the above inequality is rewritten in the following way,

$$P[F(K^*, L^*)]\cdot F(K^*, L^*) \geqq \frac{\partial P[F(K^*, L^*)]\cdot F(K^*, L^*)}{\partial K}\cdot K^* + \frac{\partial P[F(K^*, L^*)]\cdot F(K^*, L^*)}{\partial L}\cdot L^* \quad \cdots\cdots(10)$$

From (8)-a and c, we have the following equations: Let $P[F(K, L)]\cdot F(K, L)$ *be* H.

$$H_K - r = \lambda^*(H_K - s) \qquad \cdots\cdots(11)$$

$$\lambda^* s K^* = \lambda^* H - \lambda^* w L^* \qquad \cdots\cdots(12)$$

By combining (11) and (12), we have

$$(H_K - r) \cdot K^* = \lambda^* \cdot H_K \cdot K^* - \lambda^* \cdot H + \lambda^* \cdot W \cdot L^*,$$

From (10) and $\lambda \geqq 0$, we have

$$(H_K - r) \cdot K^* \leqq 0, \ or \ H_K - r \leqq 0$$

Therefore, $H_K - s \leqq 0$ must be held.

Since we have assumed $s \underline{\diagdown} r$, $H_K - s = 0$ cannot be held.

So,

$$1 = \lambda^* \frac{(H_K - s)}{(H_K - r)} \text{ and from } s \underline{\diagdown} r,\ 1 \angle \frac{H_K - s}{H_K - r}\ ;\ \lambda^* \angle 1 \text{ is concluded.}$$

Under the "fair rate of return" criterion, profit maximizing K^* and L^* must satisfy: …

$$\frac{F_K}{F_1} = \frac{r}{w} - \frac{\lambda(s - r)}{(1 - \lambda^*)\ w} \angle \frac{r}{w} \qquad \cdots\cdots(13)$$

The amount of K (capital) used, is larger than the situation under no constraints. Namely, the (13) means that the capital is excessively spent, comparing with the cost minimizing combination of productive factors in equilibrium position.

This phenomena are called "Averch–Johnson Effect." They also analyse the multi–market cases.

Public Utility can also enter other regulated markets, and the regulatory agency bases its "fair rate of return" criterion on the firm's over–all value of plant and equipment for all markets taken together, rather than computing a separate rate of return for each market.

In this case, the firm may have an incentive to enter other markets, even if the cost of so doing exceeds the additional revenues.

Expanding into other markets may enable the firm to inflate its rate-base to satisfy the constraint, and permit it to earn a greater total constrained profit.

Let the production function of the output in market 1 be

$$F^1(K^1, L^1)$$

and that in the market 2 be

$$F^2(K^2, L^2)$$

Let the prices of K and L be some constraints r and w respectively.

Suppose the firm is monopoly in market 1, and the expansion path in market 2 yields break even, i.e.

$P^2 \cdot [F^2(K^2, L^2)] - rK^2 - wL^2 = 0$ for any K^2, L^2 on the expansion path.

If the "fair rate of return" is the same for the two markets, the constraint is

$$P^1 \cdot [F^1(K^1, L^1)] \cdot F^1(K^1, L^1) - sK^1 - wL^1 + P^2 \cdot F^2(K^2, L^2) - sK^2 - wL^2 \leqq 0 \quad \cdots\cdots(14)$$

Let $F^1(K^{1*}, L^{1*})$ be the profit maximizing quantity of output without constraint, and let the amount of the profit be π^*.

That with the constraint is then written as π^{**}. Clearly $\pi^* \geqq \pi^{**}$ most be hold.

Suppose

$$P^1[F^1(K^{1*}, L^{1*})] \cdot F^1(K^{1*}, L^{1*}) - sK^{1*} - wL^{1*} = m, \ m \geqq 0 \cdots(15)$$

If, adjust

$$P^2[F^2(K^2, L^2)] \cdot F^2(K^2, L^2) - sK^2 - wL^2 = -m \quad \cdots\cdots(16)$$

the constraint is satisfied.

In addition, the firm can earn excessive profits $\pi^* - \pi^{**}$. If the expansion path of the second market yields

$$P^2 \cdot [F^2(K^2, L^2)] - rK^2 - wL^2 \geqq -(\pi^* - \pi^{**}),$$

the firm has an advantage even with losses in market 2.

Nextly, the relation between the regulation and social welfare is analysed. Consider the single market case. Let $s \gtrless r$. We first obtain the value of $\frac{dK^*}{ds}$, $\frac{dL^*}{ds}$, and $\frac{dF(K^*. L^*)}{ds}$.

Since, from (8)−a and −b, $\lambda^*=0$ implies no regulation, $\lambda^* \gtrless 0$ must hold, with the regulation.

Hence, $P[F(K^*, L^*)] \cdot F(K^*, L^*) - sK^* - wL^* = 0$.

By differentiating this equation with 's', we have

$$(P+P'F) \cdot (F_K \cdot \frac{dK^*}{ds} + F_l \cdot \frac{dL^*}{ds}) - K^* - s \cdot \frac{dK^*}{ds} - w \cdot \frac{dL^*}{ds} = 0. \quad \cdots(17)$$

But from (8)−b,

$[(P+P'F)F_l - w](1-\lambda^*) = 0$. We know $0 \angle \lambda^* \angle 1$,

so, $(P+P'F)F_l - w = 0$. $\cdots\cdots(8')-b$.

Hence, $[(P+P'F)F_K - s]\frac{dK^*}{ds} = K^*$

From (8)−a, (10) and $s \gtrless r$, we know that $(P+P'F) \cdot F_K - s \angle 0$.

Thus, we have $\frac{dK^*}{ds} \angle 0$.

Differentiating $(8')-b$ with respect to 's' yields,

$$\frac{d(P+P'F)}{dF} \cdot F_l \cdot F_K \frac{dK^*}{ds} + \frac{d(P+P'F)}{dF} \cdot F_l \cdot F_l \frac{dL^*}{ds}$$

$$+ (P+P'F) \cdot F_{lK} \cdot \frac{dK^*}{ds} + (P+P'F) \cdot F_{ll} \frac{dL^*}{ds} =$$

$$\frac{d(P+P'F)}{dF} \cdot F_l \cdot F_K + (P+P'F) \cdot F_{lK}] \frac{dK^*}{ds}$$

$$+ [\frac{d(P+P'F)}{dF} \cdot F_l^2 + (P+P'F) \cdot F_{ll}] \frac{dL^*}{ds} = 0 \quad \cdots\cdots(18)$$

If the terms in the brackets are negative, we have $\frac{dL^*}{ds} \angle 0$. It should be noted that $F_{11} \gtrless 0$ can be held with $\frac{dL^*}{ds} \angle 0$. Since $\frac{dF(K, L)}{ds} = (F_K \frac{dK}{ds}$

$+F_l \frac{dL}{ds}$), under the assumption made $\frac{dF(K^*, L^*)}{ds} \angle 0$.

Sheshinski, E.[5)*] applied these results to a social welfare function $U=U[F(K, L), K, L]$, where $U_1 \searrow 0$, $U_2 \angle 0$, $U_3 \angle 0$.

$$Let \frac{dU}{ds}=U_1 [F_K\frac{dK}{ds}+F_l\frac{dL}{ds}]+U_2\frac{dK}{ds}+U_3\frac{dL}{ds}$$

If we consider social indifference curve, utility maximizing yields:

$$\frac{U_1}{U_2}=-\frac{P}{r} \text{ and } \frac{U_1}{U_3}=-\frac{P}{w}. \qquad \cdots\cdots(19)$$

If 's' is at the unconstrained level, $\lambda^*=0$ from the previous discussion.

Then, from (8)−a and −b, $(P+P'F)F_K=r$, and $(P+P'F)F_l=w$. By using these results, $\frac{dU}{ds}$ can be rewritten as

$$\frac{dU}{ds}=\frac{U_1}{P}[(PF_K-r)\cdot\frac{dK}{ds}+(PF_l-w)\cdot\frac{dL}{ds}]=-\frac{U_1}{P}P'F(F_K\cdot\frac{dK}{ds}+F_l\cdot\frac{dL}{ds}). \qquad \cdots\cdots(20)$$

Since $P' \angle 0$ by assumption, $\frac{dU}{ds} \angle 0$ from the previous results. Therefore, if s decreases to "Active" level, social welfare increases.

The necessary condition for the maximum is given by $\frac{dU}{ds}=0$, or

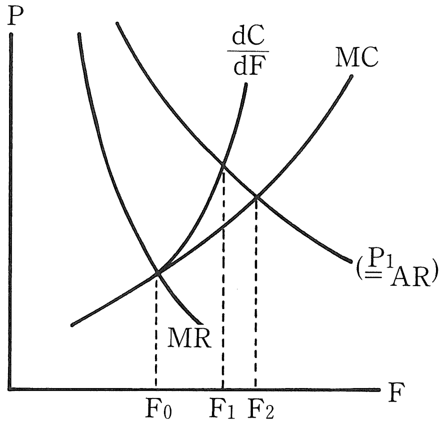
$$\frac{U_1}{P}[(P\cdot F_K-r)\frac{dK}{ds}+(P\cdot F_l-w)\frac{dL}{ds}]$$

$$=\frac{U_1}{P}[P\cdot(F_K\cdot\frac{dK}{ds}+F_l\cdot\frac{dL}{ds})-(r\cdot\frac{dK}{dK}\cdot\frac{dK}{ds}+w\cdot\frac{dL}{dL}\cdot\frac{dL}{ds})]=0 \qquad \cdots\cdots(21)$$

Or, $P\cdot\frac{dF(K, L)}{ds}=\frac{dC}{ds}$, where $C=rK+wL$, i.e. Total Costs.

We can conclude $P=\frac{dC}{dF}$ $\cdots$(22). The price is equal to the marginal cost.

Under this regulation, social welfare is maximized with P=MC. As 's' decreases with regulation, costs increase. The change in costs for a unit change in output is drawn by the curve dC/dF of Figure 1. which is higher throughout than the MC curve. The optimal 's' is set at the level corresponding to the point where dC/dF intersects the demand curve, at output F_1.



Under the "Fair rate of return criterion", social welfare is maximized at the level of output which is higher than that of unconstrained case, and lower than the Pareto optimum output F_2.

1*Averch, H. and Johnson, L.; "Behavior of the firm under Regulatory Constraint." The American Economic Review. December 1962.
2*Takayama, A.: "Behavior of the Firm Under Regulatory Constraints." The American Economic Review, June 1969.
3*Arrow, K. J. and Hurwicz, L.: "Studies in Resource Allocation Processes." N. Y. Cambridge University Press, 1977.
4*Murata, Y.: "Mathematics for Stability and Optimization of Economic Systems." N. Y. Academic Press. 1977.
5*Sheshinski, E.; "Welfare Aspects of a Regulatory Constraint: Note." American Economic Review, March (1971). p.177.